Topic 3 – Surds (Solutions)

Q1, (Jan 2006, Q8)

(i) Simplify $5\sqrt{8}+4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. [2]

(ii) Express $\dfrac{\sqrt{3}}{6-\sqrt{3}}$ in the form $p+q\sqrt{3}$, where p and q are rational. [3]

i/ $5\sqrt{8}+4\sqrt{50} = 5\sqrt{4}\sqrt{2}+4\sqrt{25}\sqrt{2} = 10\sqrt{2}+20\sqrt{2} = \boxed{30\sqrt{2}}$

ii/ $\dfrac{\sqrt{3}}{6-\sqrt{3}} \times \dfrac{6+\sqrt{3}}{6+\sqrt{3}} = \dfrac{6\sqrt{3}+3}{36\cancel{+6\sqrt{3}}\cancel{-6\sqrt{3}}-3} = \dfrac{6\sqrt{3}+3}{33} = \dfrac{2\sqrt{3}+1}{11} = \boxed{\dfrac{2}{11}\sqrt{3}+\dfrac{1}{11}}$

Q2, (Jun 2006, Q7)

(i) Simplify $6\sqrt{2}\times5\sqrt{3}-\sqrt{24}$. [2]

(ii) Express $(2-3\sqrt{5})^2$ in the form $a+b\sqrt{5}$, where a and b are integers. [3]

i/ $6\sqrt{2}\times5\sqrt{3}-\sqrt{24} = 30\sqrt{6}-\sqrt{4}\sqrt{6} = 30\sqrt{6}-2\sqrt{6} = \boxed{28\sqrt{6}}$

ii/ $(2-3\sqrt{5})(2-3\sqrt{5}) = 4-6\sqrt{5}-6\sqrt{5}+9\sqrt{5}\sqrt{5} = \boxed{49-12\sqrt{5}}$

Q3, (Jan 2007, Q7)

You are given that $a=\dfrac{3}{2}$, $b=\dfrac{9-\sqrt{17}}{4}$ and $c=\dfrac{9+\sqrt{17}}{4}$. Show that $a+b+c=abc$. [4]

$a+b+c = \dfrac{3}{2}+\dfrac{9-\sqrt{17}}{4}+\dfrac{9+\sqrt{17}}{4}$

$= \dfrac{6+9\cancel{-\sqrt{17}}+9\cancel{+\sqrt{17}}}{4} = \dfrac{24}{4} = \boxed{6}$

$abc = \dfrac{3}{2}\times\dfrac{9-\sqrt{17}}{4}\times\dfrac{9+\sqrt{17}}{4} = \dfrac{3(9-\sqrt{17})(9+\sqrt{17})}{32}$

$= \dfrac{3(81\cancel{+9\sqrt{17}}\cancel{-9\sqrt{17}}-17)}{32} = \dfrac{3(64)}{32} = 3(2) = \boxed{6}$

$\boxed{\therefore\ a+b+c = abc}$

Q4, (Jun 2007, Q8)

(i) Simplify $\sqrt{98}-\sqrt{50}$. [2]

(ii) Express $\dfrac{6\sqrt{5}}{2+\sqrt{5}}$ in the form $a+b\sqrt{5}$, where a and b are integers. [3]

i/ $\sqrt{98}-\sqrt{50} = \sqrt{49}\sqrt{2}-\sqrt{25}\sqrt{2} = 7\sqrt{2}-5\sqrt{2} = \boxed{2\sqrt{2}}$

ii/ $\dfrac{6\sqrt{5}}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}} = \dfrac{12\sqrt{5}-30}{4-\cancel{2\sqrt{5}}+\cancel{2\sqrt{5}}-5} = \dfrac{12\sqrt{5}-30}{-1} = \boxed{30-12\sqrt{5}}$

Q5, (Jan 2008, Q8)

(i) Write $\sqrt{48}+\sqrt{3}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. [2]

(ii) Simplify $\dfrac{1}{5+\sqrt{2}}+\dfrac{1}{5-\sqrt{2}}$. [3]

i/ $\sqrt{48}+\sqrt{3} = \sqrt{16}\sqrt{3}+\sqrt{3} = 4\sqrt{3}+\sqrt{3} = \boxed{5\sqrt{3}}$

ii/ $\dfrac{1}{5+\sqrt{2}}+\dfrac{1}{5-\sqrt{2}} = \dfrac{5-\cancel{\sqrt{2}}+5+\cancel{\sqrt{2}}}{(5+\sqrt{2})(5-\sqrt{2})} = \dfrac{10}{25-\cancel{5\sqrt{2}}+\cancel{5\sqrt{2}}-2}$

$= \boxed{\dfrac{10}{23}}$

Q6, (Jun 2008, Q7)

(i) Express $\dfrac{1}{5+\sqrt{3}}$ in the form $\dfrac{a+b\sqrt{3}}{c}$, where a, b and c are integers. [2]

(ii) Expand and simplify $(3-2\sqrt{7})^2$. [3]

i/ $\dfrac{1}{(5+\sqrt{3})} \times \dfrac{5-\sqrt{3}}{(5-\sqrt{3})} = \dfrac{5-\sqrt{3}}{25-\cancel{5\sqrt{3}}+\cancel{5\sqrt{3}}-3} = \boxed{\dfrac{5-\sqrt{3}}{22}}$

ii/ $(3-2\sqrt{7})(3-2\sqrt{7}) = 9-6\sqrt{7}-6\sqrt{7}+28 = \boxed{37-12\sqrt{7}}$

Q7, (Jan 2009, Q10)

(i) Express $\sqrt{75}+\sqrt{48}$ in the form $a\sqrt{3}$. **[2]**

(ii) Express $\dfrac{14}{3-\sqrt{2}}$ in the form $b+c\sqrt{d}$. **[3]**

i/ $\sqrt{75}+\sqrt{48} = \sqrt{25}\sqrt{3}+\sqrt{16}\sqrt{3} = 5\sqrt{3}+4\sqrt{3} = \boxed{9\sqrt{3}}$

ii/ $\dfrac{14}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{42+14\sqrt{2}}{9+\cancel{3\sqrt{2}}-\cancel{3\sqrt{2}}-2} = \dfrac{42+14\sqrt{2}}{7} = \boxed{6+2\sqrt{2}}$

Q8, (Jun 2009, Q8)

(i) Simplify $\dfrac{\sqrt{48}}{2\sqrt{27}}$. **[2]**

(ii) Expand and simplify $(5-3\sqrt{2})^2$. **[3]**

i/ $\dfrac{\sqrt{48}}{2\sqrt{27}} = \dfrac{\sqrt{16}\sqrt{3}}{2\sqrt{9}\sqrt{3}} = \dfrac{4\sqrt{3}}{6\sqrt{3}} = \boxed{\dfrac{2}{3}}$

ii/ $(5-3\sqrt{2})(5-3\sqrt{2}) = 25 - 15\sqrt{2} - 15\sqrt{2} + 9\sqrt{2}\sqrt{2}$

$= 25 - 30\sqrt{2} + 18 = \boxed{43 - 30\sqrt{2}}$

Q9, (Jun 2010, Q5)

(i) Express $\sqrt{48}+\sqrt{27}$ in the form $a\sqrt{3}$. **[2]**

(ii) Simplify $\dfrac{5\sqrt{2}}{3-\sqrt{2}}$. Give your answer in the form $\dfrac{b+c\sqrt{2}}{d}$. **[3]**

i/ $\sqrt{48} + \sqrt{27} = \sqrt{16}\sqrt{3} + \sqrt{9}\sqrt{3} = 4\sqrt{3} + 3\sqrt{3} = \boxed{7\sqrt{3}}$

ii/ $\dfrac{5\sqrt{2}}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{15\sqrt{2} + 10}{9 + \cancel{3\sqrt{2}} - \cancel{3\sqrt{2}} - 2} = \boxed{\dfrac{10 + 15\sqrt{2}}{7}}$

Q10, (Jan 2011, Q7)

(i) Express $\dfrac{81}{\sqrt{3}}$ in the form 3^k. **[2]**

(ii) Express $\dfrac{5+\sqrt{3}}{5-\sqrt{3}}$ in the form $\dfrac{a+b\sqrt{3}}{c}$, where a, b and c are integers. **[3]**

i/ $\dfrac{81}{\sqrt{3}} = \dfrac{3^4}{3^{\frac{1}{2}}} = 3^{4-\frac{1}{2}} = \boxed{3^{\frac{7}{2}}}$

ii/ $\dfrac{5+\sqrt{3}}{5-\sqrt{3}} \times \dfrac{5+\sqrt{3}}{5+\sqrt{3}} = \dfrac{25 + 5\sqrt{3} + 5\sqrt{3} + 3}{25 + \cancel{5\sqrt{3}} - \cancel{5\sqrt{3}} - 3} = \dfrac{28 + 10\sqrt{3}}{22}$

$= \boxed{\dfrac{14 + 5\sqrt{3}}{11}}$

Q11, (Jan 2012, Q4)

(i) Expand and simplify $(7+3\sqrt{2})(5-2\sqrt{2})$. **[3]**

(ii) Simplify $\sqrt{54}+\dfrac{12}{\sqrt{6}}$. **[2]**

i/ $(7+3\sqrt{2})(5-2\sqrt{2}) = 35 - 14\sqrt{2} + 15\sqrt{2} - 6(2)$

$= \boxed{23+\sqrt{2}}$

ii/ $\sqrt{54} = \sqrt{9}\sqrt{6} = 3\sqrt{6}$

$\dfrac{12}{\sqrt{6}} = \dfrac{12}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{12\sqrt{6}}{6} = 2\sqrt{6}$

$\Rightarrow \sqrt{54}+\dfrac{12}{\sqrt{6}} = 3\sqrt{6}+2\sqrt{6} = \boxed{5\sqrt{6}}$

Q12, (Jun 2012, Q5)

(i) Simplify $\dfrac{10(\sqrt{6})^3}{\sqrt{24}}$. **[3]**

(ii) Simplify $\dfrac{1}{4-\sqrt{5}}+\dfrac{1}{4+\sqrt{5}}$. **[2]**

i/ $\dfrac{10(\sqrt{6})^3}{\sqrt{24}} = \dfrac{10\sqrt{6}\sqrt{6}\cancel{\sqrt{6}}}{\sqrt{4}\cancel{\sqrt{6}}} = \dfrac{10(6)}{2} = \boxed{30}$

ii/ $\dfrac{1}{4-\sqrt{5}} + \dfrac{1}{4+\sqrt{5}} = \dfrac{4+\cancel{\sqrt{5}}+4-\cancel{\sqrt{5}}}{(4-\sqrt{5})(4+\sqrt{5})} = \dfrac{8}{16+\cancel{4\sqrt{5}}-\cancel{4\sqrt{5}}-5}$

$= \boxed{\dfrac{8}{11}}$

Q13, (Jan 2013, Q7)

(i) Express $\sqrt{48}+\sqrt{75}$ in the form $a\sqrt{b}$, where a and b are integers. **[2]**

(ii) Simplify $\dfrac{7+2\sqrt{5}}{7+\sqrt{5}}$, expressing your answer in the form $\dfrac{a+b\sqrt{5}}{c}$, where a, b and c are integers. **[3]**

i/ $\sqrt{48}+\sqrt{75} = \sqrt{16}\sqrt{3}+\sqrt{25}\sqrt{3} = 4\sqrt{3}+5\sqrt{3} = \boxed{9\sqrt{3}}$

ii/ $\dfrac{7+2\sqrt{5}}{7+\sqrt{5}} \times \dfrac{7-\sqrt{5}}{7-\sqrt{5}} = \dfrac{49-7\sqrt{5}+14\sqrt{5}-2(5)}{49-\cancel{7\sqrt{5}}+\cancel{7\sqrt{5}}-5}$

$= \boxed{\dfrac{39+7\sqrt{5}}{44}}$

Q14, (Jun 2013, Q7)

(i) Express $125\sqrt{5}$ in the form 5^k. **[2]**

(ii) Simplify $10 + 7\sqrt{5} + \dfrac{38}{1 - 2\sqrt{5}}$, giving your answer in the form $a + b\sqrt{5}$. **[3]**

i/ $125\sqrt{5} = 5^3 \times 5^{\frac{1}{2}} = \boxed{5^{\frac{7}{2}}}$

ii/ $\dfrac{38}{1-2\sqrt{5}} \times \dfrac{1+2\sqrt{5}}{1+2\sqrt{5}} = \dfrac{38 + 76\sqrt{5}}{1 - \cancel{2\sqrt{5}} + \cancel{2\sqrt{5}} - 4(5)} = \dfrac{38 + 76\sqrt{5}}{-19}$

$= -2 - 4\sqrt{5}$

$10 + 7\sqrt{5} - 2 - 4\sqrt{5} = \boxed{8 + 3\sqrt{5}}$

Q15, (Jun 2014, Q4)

(i) Expand and simplify $(7 - 2\sqrt{3})^2$. **[3]**

(ii) Express $\dfrac{20\sqrt{6}}{\sqrt{50}}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. **[2]**

i/ $(7 - 2\sqrt{3})(7 - 2\sqrt{3}) = 49 - 14\sqrt{3} - 14\sqrt{3} + 4(3)$

$= \boxed{61 - 28\sqrt{3}}$

ii/ $\dfrac{20\sqrt{6}}{\sqrt{50}} = \dfrac{20\sqrt{6}}{\sqrt{25}\sqrt{2}} = \dfrac{20\sqrt{6}}{5\sqrt{2}} = \boxed{4\sqrt{3}}$

Q16, (Jun 2015, Q6)

(i) Expand and simplify $(3 + 4\sqrt{5})(3 - 2\sqrt{5})$. **[3]**

(ii) Express $\sqrt{72} + \dfrac{32}{\sqrt{2}}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. **[2]**

i/ $(3 + 4\sqrt{5})(3 - 2\sqrt{5}) = 9 - 6\sqrt{5} + 12\sqrt{5} - 8(5)$

$= \boxed{-31 + 6\sqrt{5}}$

ii/ $\sqrt{72} + \dfrac{32}{\sqrt{2}} = \sqrt{36}\sqrt{2} + \dfrac{32}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = 6\sqrt{2} + \dfrac{32\sqrt{2}}{2} = 6\sqrt{2} + 16\sqrt{2} = \boxed{22\sqrt{2}}$